felt that the present theories, such as shown in Fig. 3, are adequate for the prediction of stagnation point heating during planetary re-entry at flight velocities up to the order of 35,000 to 40,000 fps.

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Thermal Ionization behind Strong Shock Waves

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Introduction

THIS paper is concerned with the flow of an ideal ionizing gas[1, 2] through a strong normal shock wave under nonequilibrium conditions. The present analysis is similar to that used by Freeman[3] for the flow of an ideal dissociating gas through a strong normal shock wave. The Bray and Wilson equation[4] for the net rate of ionization is used so that the approach to a final equilibrium ionization level behind the shock wave can be predicted. The concept of different temperatures for atoms (ions) and electrons is taken into account by considering the conservation of energy for electrons. The present paper thus formulates the strong shock problem for the ideal ionizing gas so as to allow one to determine the nonequilibrium flow properties and, in particular, the nature of the variation of the two temperatures as they approach equilibrium.

Flow through a Normal Shock Wave

In order to determine the variation of the shock parameters in the nonequilibrium region behind the shock, it is necessary to formulate the conservation laws for an ideal ionizing gas. In a coordinate system moving with the shock front, the flow is steady. Thus, the fluxes of mass, momentum, and energy across any plane behind the shock front must be equal to the same fluxes across a similar plane ahead of the shock wave. Using the concept of the ideal ionizing mon-

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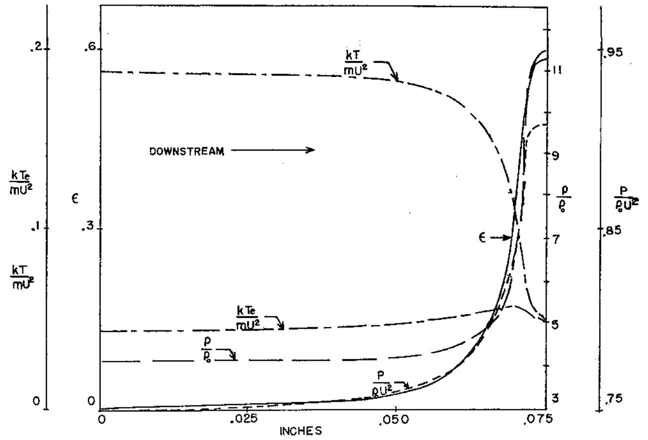

Fig. 1 Results of two-temperature analysis for strong shock ($\lambda = 1$ and $\rho_0/\rho_{ion} = 1.7 \times 10^{-7}$).

atomic gas,[1, 2] these three conservation equations can be written for strong shock waves as follows:

Mass

$$\rho u = \rho_0 U \tag{1}$$

Momentum

$$p + \rho u^2 = \rho_0 U^2 \tag{2}$$

Energy

$$i + (u^2/2) = U^2/2 \tag{3}$$

where the enthalpy of the gas is

$$i = \tfrac{5}{2}(k/m)[T + \epsilon T_e] + (k/m)T_{ion}\epsilon \tag{4}$$

In these equations, ρ_0 and U are the given values of density and velocity ahead of the shock, and p, ρ, and u are the pressure, density, and velocity behind the shock front. In Eq. (4), k is Boltzmann's constant, m the mass of the atom, T the temperature of the atoms and ions, T_e the electron temperature, ϵ the degree of ionization, and T_{ion} the ionization potential of the gas in degrees Kelvin. In the region behind the Rankine-Hugoniot shock front, the gas is assumed to be inviscid and without heat conduction and radiation energy flux.

The equation of state is

$$p/\rho = (k/m)[T + \epsilon T_e] \tag{5}$$

Two additional equations are required to specify the thermodynamic quantities completely, namely, an equation for the net rate of ionization and an equation relating the two temperatures T and T_e.

Since the flow is steady, the changes are purely convective, and the equation for the net rate of ionization in the nonequilibrium region can be written in the form[4]

$$u\frac{d\epsilon}{dx} = A\rho^*\epsilon(1-\epsilon)T_e^{*3/2}\left(\frac{T_{exc}}{T_e}+2\right)\exp\left(-\frac{T_{exc}}{T_e}\right) - A\rho^{*2}\epsilon^3\left(\frac{T_{exc}}{T_e}+2\right)\exp\left(\frac{T_{ion}}{T_e}-\frac{T_{exc}}{T_e}\right) \tag{6}$$

where x is a coordinate measured normal to the shock wave in the downstream direction and u is the velocity of the gas in this direction. In Eq. (6), T_{exc} is the energy of the first excited state of the gas expressed in degrees Kelvin, A is a constant (for argon, $A = 3.349 \times 10^{16}$ sec^{-1}), and the dimensionless density ρ^* and temperature T_e^* are defined as[2]

$$\rho^* = \rho/\rho_{ion} \qquad T_e^* = T_e/T_{ion} \tag{7}$$

In Eq. (7), ρ_{ion} is the characteristic density and, for argon, has the value $\rho_{ion} = 2.9326 \times 10^2$ slugs/ft^3.

At some distance downstream of the shock which is to be determined, the equilibrium ionization level ϵ_{eq} is reached. Under equilibrium conditions $ud\epsilon/dx = 0$, $\epsilon = \epsilon_{eq}$, and $T \equiv T_e$; thus Eq. (6) reduces to the well-known Saha equation.

The remaining equation relating the two temperatures T and T_e obtained by considering the conservation of energy for electrons is[4]

In general, it is only necessary to specify the shock strength λ, the freestream density parameter ρ_0/ρ_{ion}, and the gas. For the results shown in Fig. 1, the shock strength was specified as $\lambda = 1$, and the freestream density parameter was chosen as $\rho_0/\rho_{ion} = 1.7 \times 10^{-7}$. These given conditions, used in conjunction with Saha's equation and Eqs. (1, 10, and 12), yield the downstream equilibrium ionization level $\epsilon_{eq} = 0.59$. The initial degree of ionization was taken as $\epsilon_i = 0.001$.

$$T^* = T_e^* + \frac{K(1 + \frac{3}{2}T_e^*)T_e^{*3}[(T_{exc}/T_e) + 2]\exp(-T_{exc}/T_e)[(1 - \epsilon) - \rho^*\epsilon^2 T_e^{*-3/2} e^{1/T_e^*}]}{\epsilon \ln(DT_e^{*3}/\rho^*\epsilon)} \tag{8}$$

where $T^* = T/T_{ion}$, and, for argon, $K = 1.53 \times 10^4$ and $D = 0.2980$.

Two-Temperature Analysis of Nonequilibrium Region

The variation of ϵ with x in the nonequilibrium region can be found from Eqs. (6) and (1); thus,

$$x = \int_{\epsilon_i}^{\epsilon} \frac{(u/U)d\epsilon}{\frac{A}{U}\epsilon\left[\frac{T_{exc}}{T_e} + 2\right]\frac{U}{u}\frac{\rho_0}{\rho_{ion}}\left(\frac{T_e}{T_{ion}}\right)^{3/2}\exp\left[\frac{T_{ion}}{T_e} - \frac{T_{exc}}{T_e}\right]\left\{(1 - \epsilon)e^{-T_{ion}/T_e} - \epsilon^2\frac{\rho_0}{\rho_{ion}}\frac{U}{u}\left(\frac{T_{ion}}{T_e}\right)^{3/2}\right\}} \tag{9}$$

In Eq. (9), the lower limit of integration, ϵ_i, is the initial value of the degree of ionization upstream of the shock. The thickness of the nonequilibrium ionization region, x, can be defined conveniently by using $\epsilon = 0.99\epsilon_{eq}$ for the upper limit of integration. The equilibrium value of the ionization, ϵ_{eq}, is obtained from Saha's equation for the given conditions.

To facilitate the numerical integration, it is desirable to write Eq. (9) in a more convenient form. Thus, combining Eqs. (1–5), the velocity ratio can be written as

$$\frac{u}{U} = \frac{5}{8}\left\{1 - \left[1 - \left(\frac{4}{5}\right)^2\left(\frac{\lambda - \epsilon}{\lambda}\right)\right]^{1/2}\right\} \tag{10}$$

where

$$\lambda = mU^2/2kT_{ion} \tag{11}$$

It is seen that the parameter λ is a measure of the strength of the shock, since it is the ratio of the kinetic energy of the freestream to the ionization energy of the gas.

It only remains to determine the corresponding values of T_e, T, and ϵ to be used in the range of integration. This is accomplished by combining Eqs. (1, 2, and 5) to get

$$T = \frac{mU^2}{k}\left[\frac{u}{U} - \left(\frac{u}{U}\right)^2\right] - T_e\epsilon \tag{12}$$

Using Eqs. (7) and (10) and equating the right-hand sides of Eqs. (12) and (8) gives an expression relating T_e and ϵ. For a series of values of ϵ in the range of integration, corresponding values of electron temperatures T_e can thus be calculated; Eq. (12) can then be used to calculate the corresponding atom (ion) temperatures T.

The numerical integration of Eq. (9) can be carried out using the corresponding values of T_e and ϵ, the results of which give the function $x = x(\epsilon)$. That is, Eq. (9) has the form

$$x^* = \int_{\epsilon_i}^{\epsilon} F(\epsilon)d\epsilon \tag{13}$$

where the dimensionless distance

$$x^* = \frac{A(\rho_0/\rho_{ion})x}{U} = \frac{A(\rho_0/\rho_{ion})x}{(2\lambda kT_{ion}/m)^{1/2}} = \frac{A(\rho_0/\rho_{ion})x}{(2\lambda)^{1/2}v_{ion}} \tag{14}$$

The parameter v_{ion} is the characteristic velocity of the gas ($v_{ion} = 2.0209 \times 10^4$ fps for argon). Once the function $x(\epsilon)$ is known, the values of all other physical variables can be found from Eqs. (1, 5, and 10).

Results

The integration of Eq. (9) has been carried out for argon for a series of strong shock conditions. Typical results for the calculations are shown in Fig. 1.

The results show that, for the given freestream conditions, equilibrium is reached at a distance of 0.075 in. downstream of the shock. Over the first two-thirds of the nonequilibrium region, there is very little change in the shock parameters; rapid changes in pressure, density, and atom (ion) temperature occur in the latter third.

Perhaps the most interesting and surprising result is that, for all practical purposes, the electron temperature remains constant at the equilibrium value throughout the nonequilibrium region. The use of the assumption of a constant electron temperature at the equilibrium value would represent a considerable simplification in the two-temperature analysis of the nonequilibrium region behind strong shocks.

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Self-Preservation in Fully Expanded Round Turbulent Jets

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IT is well established that radial velocity profiles obtained at different axial locations in the fully developed region of subsonic, constant-density, axially symmetric, turbulent, free jets issuing into a stagnant medium can be normalized to congruent curves by plotting them in the proper reduced coordinates. Townsend's book,[1] for example, shows that, when the velocity (u) measured at a given radial distance

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